Written Exam at the Department of Economics summer 2017

Pricing Financial Assets

Final Exam

17 August 2017

(3-hour closed book exam)

Please note that the language used in your exam paper must correspond to the language for which you registered during exam registration.

This exam question consists of 2 pages in total

NB: If you fall ill during the actual examination at Peter Bangsvej, you must contact an invigilator in order to be registered as having fallen ill. Then you submit a blank exam paper and leave the examination. When you arrive home, you must contact your GP and submit a medical report to the Faculty of Social Sciences no later than seven (7) days from the date of the exam.

The Exam consists of 3 problems that will enter the evaluation with equal weights.

1. Let the time t spot exchange rate for a foreign currency be S_t, denoting the value of one unit of the foreign currency as measured in the domestic currency.

   Assume that the exchange rate can be be modelled (under the original probability measure $\mathbb{P}$) by the geometric Brownian motion
$$dS = \mu_S S dt + \sigma_S S dz$$
   where μ_S and $\sigma_S > 0$ are constants, and where dt and dz are the standard shorthand notations for a small time-step and a Brownian increment.

   Assume that the (continuously compounded) domestic and foreign risk free interest rates are constants r and r_f, respectively.

   (a) What will the drift rate of the exchange rate be under the domestic risk neutral probability measure ($\mathbb{Q}$) and a no-arbitrage assumption? Comment on the influence of the difference of domestic and foreign interest rates on the result.

   (b) Consider at time $t < T$ a forward contract on the foreign currency for payment and delivery at time T. What is the arbitrage free forward price F_t^T?

   (c) Use Ito's lemma to show that the volatility of F_t^T goes to zero as time t approaches maturity T.

2. Let S_t be the price in the domestic currency of one unit of a foreign currency. Assume that there are constant risk free, continuously compounded interest rates of r and r_f in the domestic and foreign currencies, respectively.

   Consider a derivative with price $V(S, t)$ as some function of the current exchange rate S_t and time t (and further implicit parameters).

   (a) Define and interpret the Delta, Gamma and Theta of the derivative.

   (b) Let $c(S, K, T, r, r_f)$ and $p(S, K, T, r, r_f)$ be the price at time $t = 0$ of a European call and a European put, respectively, on the currency with the same strike K and expiry T. Derive the call-put-parity.

   (c) Use the call-put-parity to find a relationship between the Deltas of the call and put. Repeat this for Gamma.

   (d) Suppose a portfolio of the foreign currency and/or derivatives on the currency is Delta-neutral, and that there are no arbitrage possibilities. Let the value of the portfolio be $\Pi(S, t)$. What can we say about the relation between the Theta and Gamma of the portfolio?

3. The LMM-model describes the evolution of the term structure of simple interest rates. Let $F_k(t)$ be the (simple) forward rate between times t_k and t_{k+1} that can be contracted for at time $t \leq t_k < t_{k+1}$, and let τ_k be the compounding period between t_k and t_{k+1}. Let $P(t, t_k)$ denote the price at t of a zero coupon bond that matures at time t_k.

   (a) What is the relationship between a particular forward rate F_k and prices of zero coupon bonds?

   (b) Assume that the forward rates are driven by one factor. What choice of numeraire will lead to the forward rate $F_k(t)$ being a martingale, e.g. of the form
$$dF_k(t) = \zeta_k(t) F_k(t) dz$$
   where dz is the standard short hand notation for the change of a Brownian motion.

   (c) In the development of the LMM-model a rolling numeraire is used. Explain in general terms how this is constructed (explicit formulas are not required, but you should discuss change of numeraires).